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THE THERMAL CONDUCTIVITY OF LIQUIDS

A THESIS

Presented to

The Faculty of the Division of Graduate Studies

by

Lee Handon Knight

In Partial Fulfillment

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THE THERMAL CONDUCTIVITY OF LIQUIDS

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SUMMARY

This investigation was conducted to design and construct a relatively simple device for use in measuring the thermal conductivity of liquids. It was also desired that the equipment permit accurately measured variation in the test liquid thickness in order to investigate the effect of liquid thickness in thermal conductivity measurements.

Design characteristics of two devices now in use were incorporated and modified to permit liquid layer thickness variations by means of a micrometer adjustment. A literature search, conducted in connection with the continuous adjustment feature, revealed no such device now in use.

Performance of the equipment was tested by measuring the thermal conductivity of water and of castor oil. Values of the thermal conductivity of these liquids were found to compare favorably with published data.

A test was made with a mineral oil specimen of 0.015 inches thickness in the temperature range of $90^{o}F$ to $190^{o}F$. The measured values of the thermal conductivity ranged from 0.093 BTU per $ft.^2$ per hr. per ^{o}F per ft. at the low temperature to 0.091 BTU per $ft.^2$ per hr. ^{o}F per ft. at the higher temperature.

An experiment was also conducted to study the effect of varying liquid layer thickness in thermal conductivity measurements. The results of this test showed that changes in the liquid thickness produced changes in the measured values of thermal conductivity at thicknesses exceeding 0.030 inches. Additional tests indicated that a portion of these changes were quite possibly a result of convective currents in the liquid surrounding the heating unit, which were manifest in the liquid gap at liquid thicknesses greater than 0.030 inches. It was also suspected that some error was incurred by the deviation of heat flux lines from a normal between the main heater and cold plate surfaces. However, the data gathered in this part of the experiment were insufficient to warrant unqualified conclusions.

CHAPTER I

INTRODUCTION

The purpose of this experiment was to construct a device for measuring the thermal conductivity of liquids in the range of $100^{\circ}F$ to $200^{\circ}F$ to an accuracy of $\pm$ 5 per cent. It was also desired to investigate the effect of liquid layer thickness on thermal conductivity measurements of liquids. A literature search revealed that an investigation of this effect was conducted by Sakiadis and Coates (1)* in 1953. However, varying the fluid thickness in the apparatus used in this experiment required additional equipment for each different fluid thickness.

Many methods have been devised for measuring the effect of liquid thickness on thermal conductivity of liquids using upward heating of the liquid. A great many of these methods are reported in the literature search by Sakiadis and Coates (2). The results of these experiments indicate that heat transfer by conduction alone can occur in fluids heated from below if the product of the Grashof Number ($N_{Gr} = \frac{g\ L^3 \beta \theta}{\nu^2}$) and the Prandtl Number ($N_{Pr} = \frac{\nu}{\alpha}$) is less than some critical value Z. There seems to be no general agreement of the observed values of Z. However, the experimental value of 1612 reached by

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*Numbers in parentheses refer to items in the Bibliography.

Mull and Reicher (3) is in close agreement with Low's (4) theoretically predicted value of 1704.4. Sakiadis and Coates (1) in their experiment used downward heating of the liquid and report that no appreciable convective currents occurred in the liquid tested in a thickness range of one to two inches. The apparatus used by these investigators is described in a later paragraph.

In view of these results, an investigation of instruments incorporating downward heating was conducted. During the course of this investigation, the bibliographies in the works of Sakiadis and Coates (5) and J. F. D. Smith (6) were valuable aids. Instruments have been constructed by Sakiadis and Coates (7), Bates (8), J. F. D. Smith (9), Kaye and Higgins (10), Jakob (11) and others. Of the units cited, only that of Sakiadis and Coates (7) had any provision for varying the fluid thickness. This instrument consisted of a lower plate cooled by water and an upper plate heated by water vapor. The test fluid was placed in an insulated glass cell with an annular rubber stopper in each end. Metal discs, which fitted into the holes in the stoppers, were soldered to the cold plate and hot plate. Thermocouples were located in the cold plate and hot plate for measurement of temperature drop through the liquid. The cell was equipped with suitably placed thermocouples for measurement of temperature gradients in the liquid. Changes in liquid thickness were accomplished by using cells of different lengths.

For the present experiment, it was desirable to construct a unit in which fluid thickness variation could be made in very small increments without disturbing the unit otherwise. To accomplish this, the instrument designed by Kaye and Higgins (10) seemed to offer the best possibility for modification. It consisted of a heating plate mounted 0.015 inches above a finned cold plate by means of three small spacers. The test fluid was held in a cup mounted concentrically to the cold plate in such a manner that the liquid filled the 0.015 inch space between the heater and cold plate. A guard heater was placed above the main heater to prevent vertical heat loss. The entire unit, with the exception of the cold plate fins, was enclosed in an oven in which the temperature was maintained at a constant $10^{\circ}F$ below that of the main heater. Corrections were made for the heat loss at the edge of the main heater caused by the $10^{\circ}F$ temperature differential.

The design described above was modified to permit adjustment of the liquid layer thickness by means of a depth micrometer as illustrated in Figure 1, page 12. A guard ring heater, suggested by Jakob's (11) design of a device for measuring thermal conductivity of non-metallic solids, was also incorporated. The use of the guard ring heater eliminated the necessity for the surrounding oven, as well as heat loss corrections for the heater edge.

CHAPTER II

INSTRUMENTATION AND EQUIPMENT

General.-- A sketch of the complete unit is shown in Figure 1, page 12. The heating unit is shown supported by rods mounted in an adjustment piston. The piston itself fits into a cylinder machined into the cold plate. The space between the heating element and the cold plate was filled with the test liquid, which was held in the liquid retaining cup. The liquid level in the retaining cup was maintained sufficiently high to insure complete filling of this gap at all times. Adjustment of the gap was accomplished by turning the depth micrometer fitted below the cold plate. The location of the thermocouples, as well as the relative position of all elements of the heating unit is shown in Figure 2, page 13. The coils shown on the cold plate are machined for counter flow of the coolant, thereby maintaining a relatively constant temperature distribution.

Heater and Guard Circuits.--The heater and guard circuits were supplied with 115 volt, 60 cycle alternating current regulated to $\pm$ 0.5 per cent. Fine voltage adjustments were accomplished by placing 3 variable transformers in series in each circuit such that a 330° turn on the final transformer varied the voltage from 0 to 35 volts. In this manner, adjustments of less than 0.100 volts could be made. A complete diagram of these circuits appears in Figure 3, page 14. Current readings in the guard circuits were recorded for general information only, using standard General Electric ammeters. Measurements of the main heater current were accomplished by reading the voltage drop across a calibrated resistor which was placed in series with the heater windings as shown in Figure 3, page 14. This resistor was constructed of Constantan wire, and had an impedence value of 3.00 ohms. The temperature-resistance coefficient of this resistor was such that thermal change was found to be negligible. The voltage drop across the resistor, as well as that across the main heater windings, was measured with a Ballantine Vacuum Tube Volt Meter calibrated with a DuMont Voltage Calibrator to an accuracy of less than 0.5 per cent of full scale deflection on a 10 volt scale.

Isothermal Surface.-- In order to maintain an even temperature distribution throughout the heater plate, and to insure complete isolation of the resistance windings, an aluminum disc 0.25 inches thick and 3.00 inches in diameter was bonded with an electrical insulating compound to the top of the heater and ring guard plate. The bonding agent acted as a void filler between the aluminum disc and the heater, preventing an uneven temperature distribution in the isothermal plates. This plate also served as a repository for two thermocouples used in balancing the temperatures of the upper guard and the main heater. The positions of the ring guard, main heater, isothermal plate, and upper guard are shown in Figure 2, page 13.

Temperature Measurement.--Nine copper-constantan thermocouples were used to measure temperature differences. Calibration of these thermocouples, one against the other, showed a maximum potential difference of 1 microvolt at 30^{o}F. This difference was considered negligible. The copper leads of the cold junction of the thermocouples were fused together so that temperature differences between any two thermocouples could be measured. A schematic of this arrangement appears in Figure 4, page 15.

Three thermocouples were placed in the main heater and ring guard plate at a distance of 1/32 inch away from the surface in contact with the test liquid. One of these was placed at the center of the heater, one at the outer edge of the heater, and one at the inner edge of the ring guard. This provided a means of checking the radial temperature gradient of the heater as well as balancing the ring guard and heater temperatures. Two thermocouples were placed in the cold plate, also 1/32 inch away from the test liquid contact surface, and directly below those in the main heater plate. The temperature drop through the test liquid was measured using the center heater thermocouple and the center cold plate thermocouple.

Two thermocouples were also placed in the isothermal plate, one at the center and one at the edge of the plate. Directly above these, two other thermocouples served as a means of balancing the main heater and upper guard heater temperatures to assure minimum heat flow between the two heaters. A Type K-3 Leeds and Northrup potentiometer was used to measure potential difference between thermocouples. Manufacturer's specification of the accuracy of this instrument is $\pm.65$ microvolts on the one millivolt scale.

A tenth thermocouple, calibrated at 32°F and 212°F, was placed in the cold plate at a distance from the center equal to the mean radius of the main heater plate. This thermocouple was used for actual temperature measurements, and temperatures throughout the system were calculated using it as the base. Positions of all the thermocouples are shown in Figure 4, page 15. All thermocouples were insulated from their respective plates by an electrical insulating compound.

CHAPTER III

CONSTRUCTION

Heater and Guard Ring Plate. --Copper, because of its very high thermal conductivity, was chosen as the most suitable material for an isothermal heater plate. The plate was machined to a diameter of 1.550 inches and a thickness of 0.37 inches. A spiral groove 1/16 inch wide and 1/16 inch deep was machined into the face of the plate. The groove was then coated with an electrical insulating compound and wound with number 24 Constantan wire as the resistance element. The resistance element was also covered with insulating compound.

The ring guard was constructed of the same material as the main heater plate and in the form of a ring of 3 inches outside diameter and 2 inches inside diameter. It was grooved, coated and wound in the same manner as the main heater. A Teflon ring was then fitted into the annulus between the main heater and the ring guard. This ring, with a thermal conductivity of approximately 0.2 BTU per $ft.^2$ per hr. per $^{\circ}F$ per ft., served as an effective thermal barrier between the heater and the ring guard. An analysis of heat loss or gain through the ring appears in Appendix B, page 33. The ring guard and Teflon insulator were then pinned to the main heater plate by means of 1/16 inch Delrin rods pressed into holes drilled at 90° intervals around the periphery. Delrin was chosen for these rods because of its relatively high strength and low thermal conductivity (K is approximately 0.1 BTU per ft.^{2} per hr. per $^{\circ}$F per ft.). Three 1/16 inch holes were drilled into the assembled plate for placement of thermocouples. The completed plate was then lapped until the ring guard, insulator, and main heater were flat to an estimated .0001 inches. The isothermal plate was then bonded to the resistance wire side of the guard-heater assembly. A sectional view of the assembled plate is shown in Figure 2, page 13.

Upper Guard Plate.--The upper guard heater plate was constructed of aluminum in the same manner as the main heater. The diameter of the plate was 3 inches, the same as the diameter of the assembled ring guard-main heater plate. Two 1/16 inch diameter holes were drilled into this plate for thermocouple placement.

Cooling Element.--The cooling element was constructed of aluminum with a cold plate surface diameter of 3 inches. This surface was lapped to an estimated flatness of .0001 inches. A Plexiglas cup with an "O" ring seal was fitted around the top of the cooling element to serve as a liquid container. This cup was deep enough to insure complete filling of the liquid gap from 0 to 0.125 inches.

Adjustment of the liquid gap was accomplished by placing a piston into a hole machines into the bottom of the cooling element. The diametral clearance between the piston and cylinder was held to less than 0.0002 inches to prevent misalignment. The piston contained three rods which were inserted into holes drilled from the cold plate surface into the piston cylinder. To insure evenness of length of the rods, the piston and rods were inserted into the cooling element and the rods lapped with the cold plate surface. A depth micrometer was then mounted to the lower end of the cooling element in such a manner that the piston moved upward as the micrometer screw was turned.

It was at first thought that the heat transfer surface of the cooling element was sufficient to provide adequate cooling of the apparatus without the use of cooling coils. However, the initial test of the thermal conductivity of a mineral oil specimen of 0.015 inches thickness vs. mean liquid temperature produced results considered to be erroneous. An inspection of the unit indicated that error could be introduced in the liquid gap thickness because of differential expansion in the cooling element in the event of temperature changes. An analysis of this effect appears in Appendix C, page 33.

To prevent excessive heating of the cooling element, cooling coils were added by machining two spiral grooves 1/4 inch wide and 1/4 inch deep around the element and connecting one end of the grooves so that counter flow of the coolant liquid occurred. A Plexiglas tube was pressed over the grooves and machined so that the bottom edge was level with the bottom of the cooling element. A brass plate, containing inlet and outlet tubes, was then fixed to the bottom of the element and all joints sealed with potting compound. A sectional view of the final version of the cooling element appears in Figure 1, page 12.

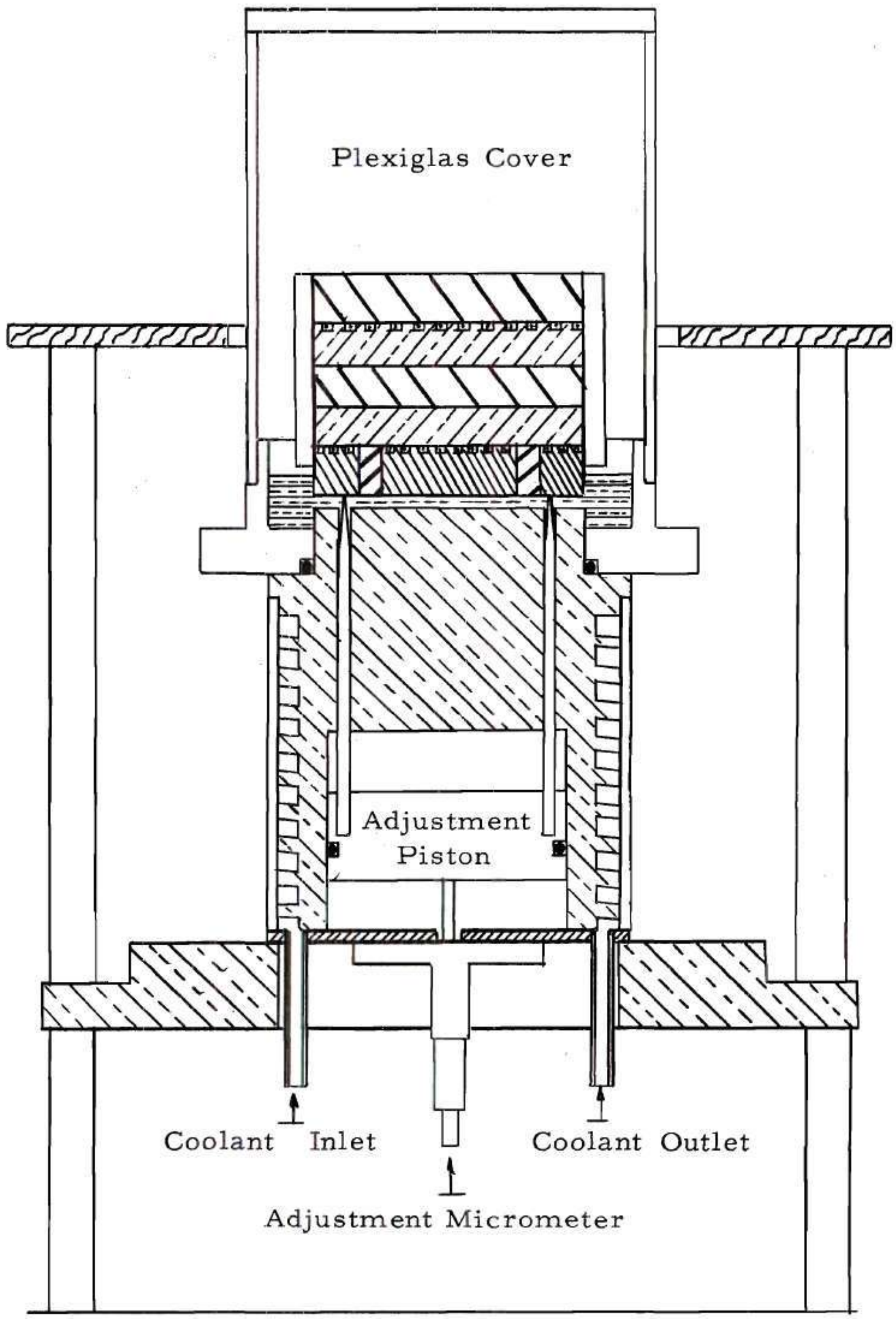

Figure 1. Sectional View of Assembled Unit.

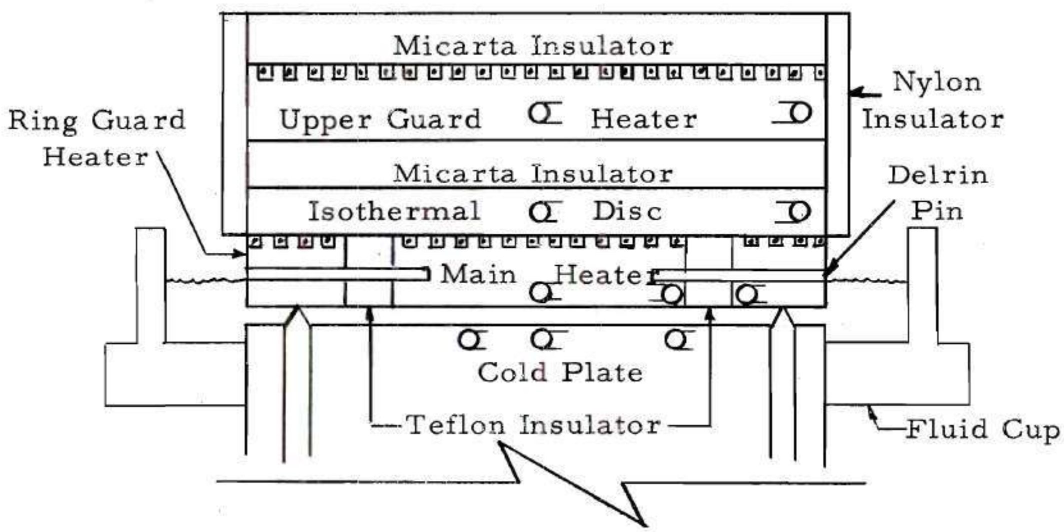

Figure 2. Sectional View of Heating Unit and Cold Plate Showing Components and Thermocouple Placement.

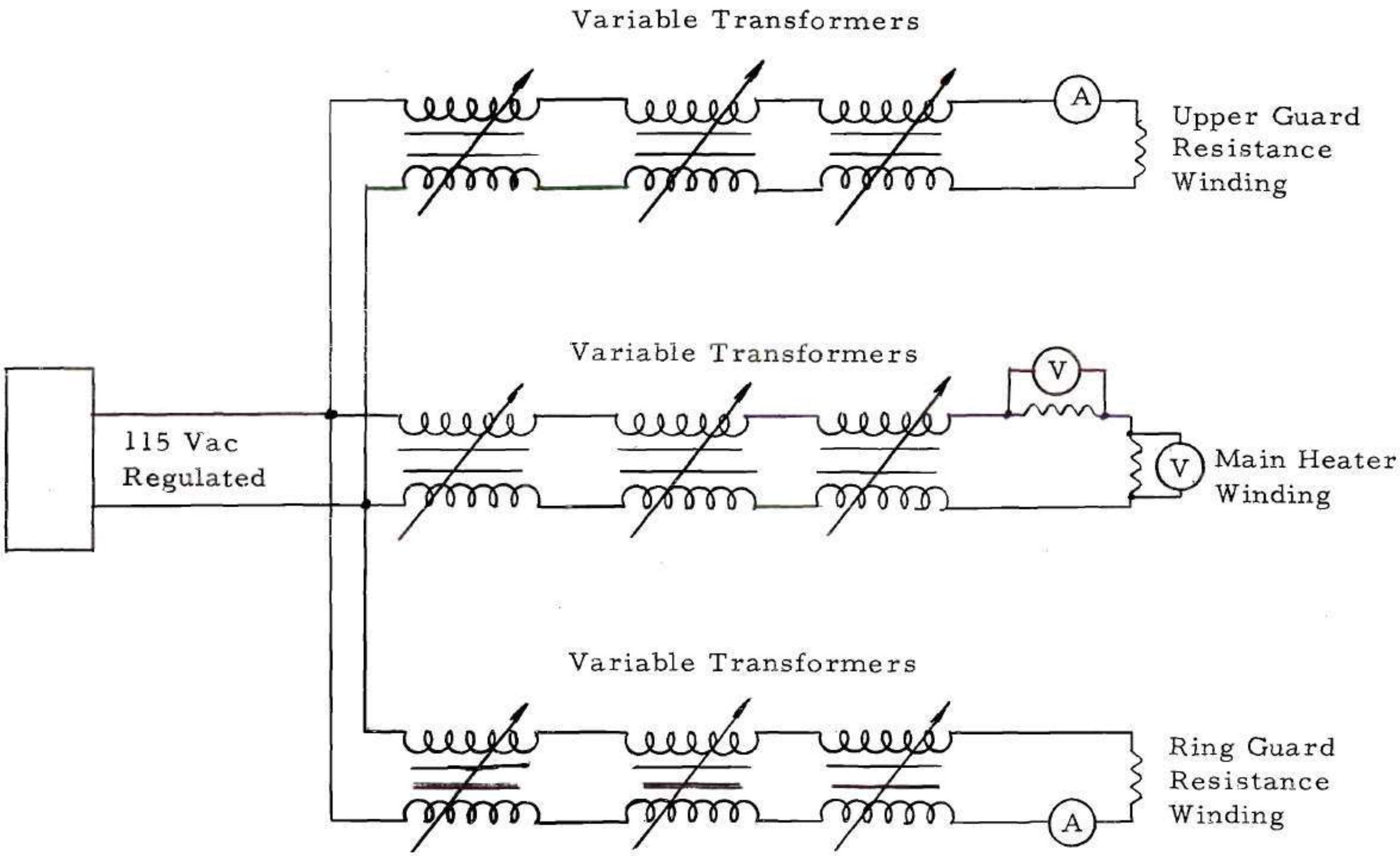

Figure 3. Schematic Wiring Diagram

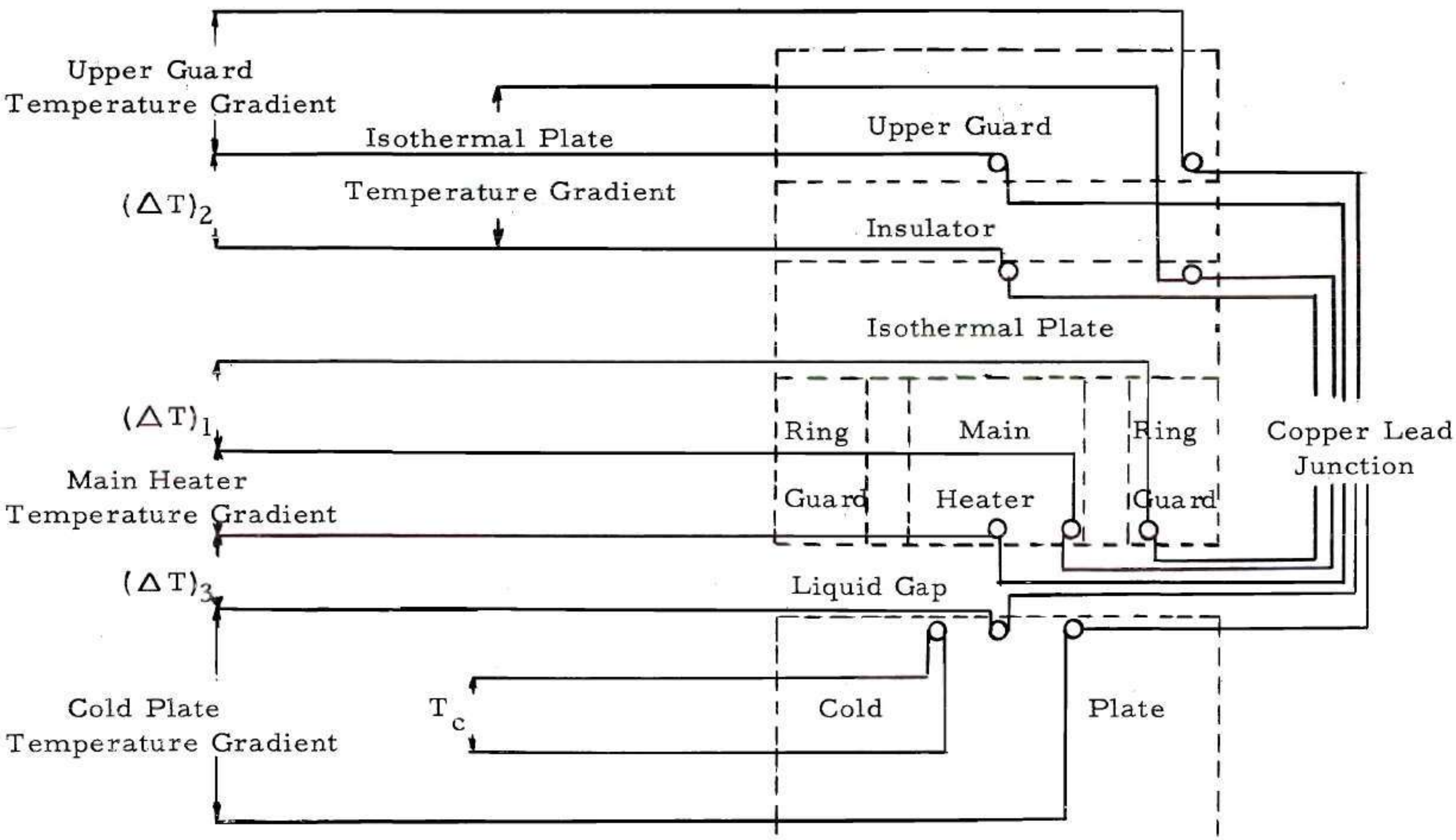

Figure 4. Schematic of Thermocouple Arrangement.

CHAPTER IV

PROCEDURE

Operation of Equipment

Constant Liquid Thickness with Variable Mean Liquid Temperature.-- In measuring the thermal conductivity of liquids, the most generally used liquid thickness has been in the order of 0.015 inch. Accordingly, micrometer adjustment for this thickness was made in this experiment by pressing the heater and ring guard plate against the cold plate, and turning the micrometer screw until contact was established between the adjustment rods and the ring guard section of the heater plate. The pressure was then relieved and the micrometer screw was then turned to extend the rods an additional 0.015 inch.

In starting a test run all heaters were placed in operation at low power settings and a small amount of water flowed through the cooling coils. It was found that approximately 4 hours were required to reach an equilibrium state. After an initial state of equilibrium was reached, adjustments were made to equalize the temperatures of the guard and main heater circuits by making changes in the guard circuit power levels. This operation was rather tedious and for this reason, temperature differences of $.25^{\circ}F$ were considered to be within the limits of accuracy. Representative time requirements for obtaining a data point are shown in Run 5, page 44.

As the power was increased, it was also necessary to increase the coolant flow in order to prevent heating and resultant differential expansion in the cooling element. A temperature change of $\pm 10^{\circ}F$ in the cooling element was considered allowable. The error involved in the temperature differences between the guards and main heater are analyzed in Appendix B, page 29.

When the main heater, ring guard, and upper guard temperatures were balanced to the limits stated and the cooling element temperature change was within the specified range, the measured values of main heater current and voltage, and the temperature drop through the liquid layer were recorded. This process was repeated in steps to a maximum mean liquid temperature of approximately $200^{\circ}F$.

<u>Constant Mean Liquid Temperature with Varying Liquid Layer Thickness.</u>--In this test, the micrometer adjustment was initially set at 0.015 inch as in the preceding case. The heater and guard circuits were placed in operation at relatively high power settings and a considerable quantity of water flowed through the cooling coils. The same procedure as outlined above was followed in balancing temperatures and recording data. In this case, however, the liquid thickness was varied while the mean liquid temperature was held as nearly constant

as possible. To maintain a constant mean liquid temperature, it was necessary to decrease both the power and the coolant flow as the liquid thickness increased.

Performance

A test of the thermal conductivity of water vs. mean liquid temperature was made to evaluate the overall performance of the equipment. Experimental values published by Maxwell (12) and Eckert (13) were used for comparison purposes. The results obtained with the instrument described above are shown in Figure 5, page 21 along with the data of Maxwell (12) and Eckert (13). A second test was run using castor oil as the test liquid. The results of this test are compared with data of Kaye and Higgins (14) in Figure 6, page 22. In both tests, the data obtained are in good agreement with that of the authors cited. A sample calculation of thermal conductivity values appear in Appendix A, page 28.

CHAPTER V

DISCUSSION OF RESULTS

Figure 5, page 21 represents the results of the performance test with water. Curves representing data published by Maxwell (12) and Eckert (13) are shown for comparison. The results of a second performance test, using castor oil as the test liquid, are shown in Figure 6, page 22. Data of Kaye and Higgins (14) are shown in this figure for comparative purposes.

Data taken in a test of the thermal conductivity of a mineral oil specimen of 0.015 inches thickness vs. mean liquid temperature appear in Figure 7, page 23. The measured values of K_m in this test ranged from 0.094 at a mean temperature of 89.5°F to 0.092 at 191.2°F.

Curve Number 1 of Figure 8, page 24 represents a plot of the thermal conductivity of mineral oil vs liquid thickness at constant mean liquid temperature. The values of K_m recorded in this test range from 0.090 BTU per ft.^{2} per hr. per °F per ft. at 0.015 inches thickness to 0.111 BTU per ft.^{2} per hr. per °F per ft. at 0.065 inches. As shown by the curve, the measured values of K_m began to increase at a liquid thickness of approximately 0.030 inches.

The experiment was repeated after adding one inch of insulating material around and above the liquid retaining cup. In this case, data was recorded in the direction of decreasing liquid thickness as well as increasing thickness. The results of this test are shown as Curves Number 2 and 3 of Figure 8, page 24. The measured values of K_m ranged from 0.091 at 0.015 inches of liquid thickness to 0.095 at 0.065 inches, showing a considerable decrease in K_m in the higher thickness ranges when compared with the data taken before adding insulation. An investigation of possible causes of this discrepancy appears in Appendix D, page 36.

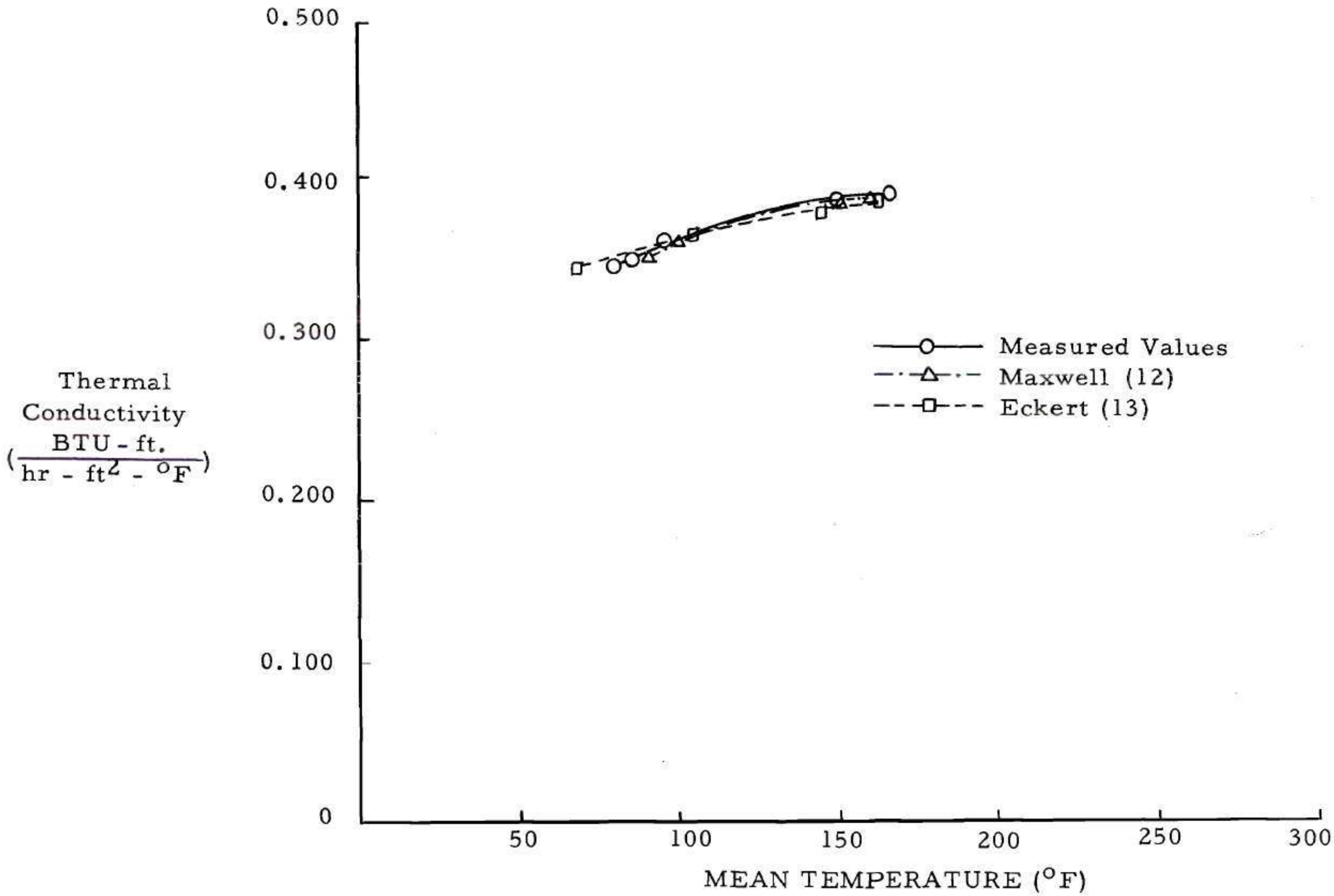

Figure 5. Comparison of the Measured Values of the Thermal Conductivity of Water with Data of Maxwell and Eckert.

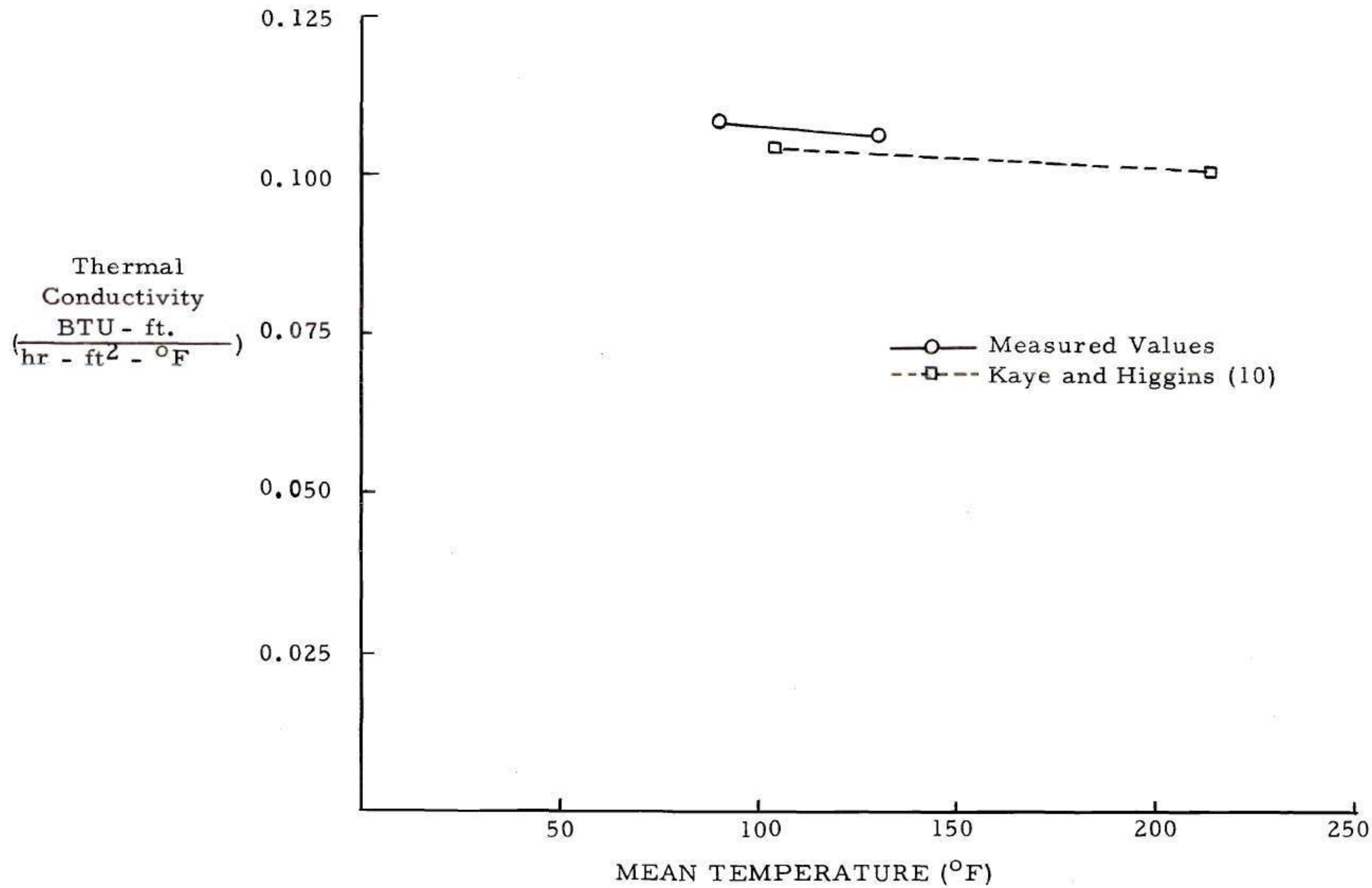

Figure 6. Comparison of Data on Castor Oil with Results of Kaye and Higgins.

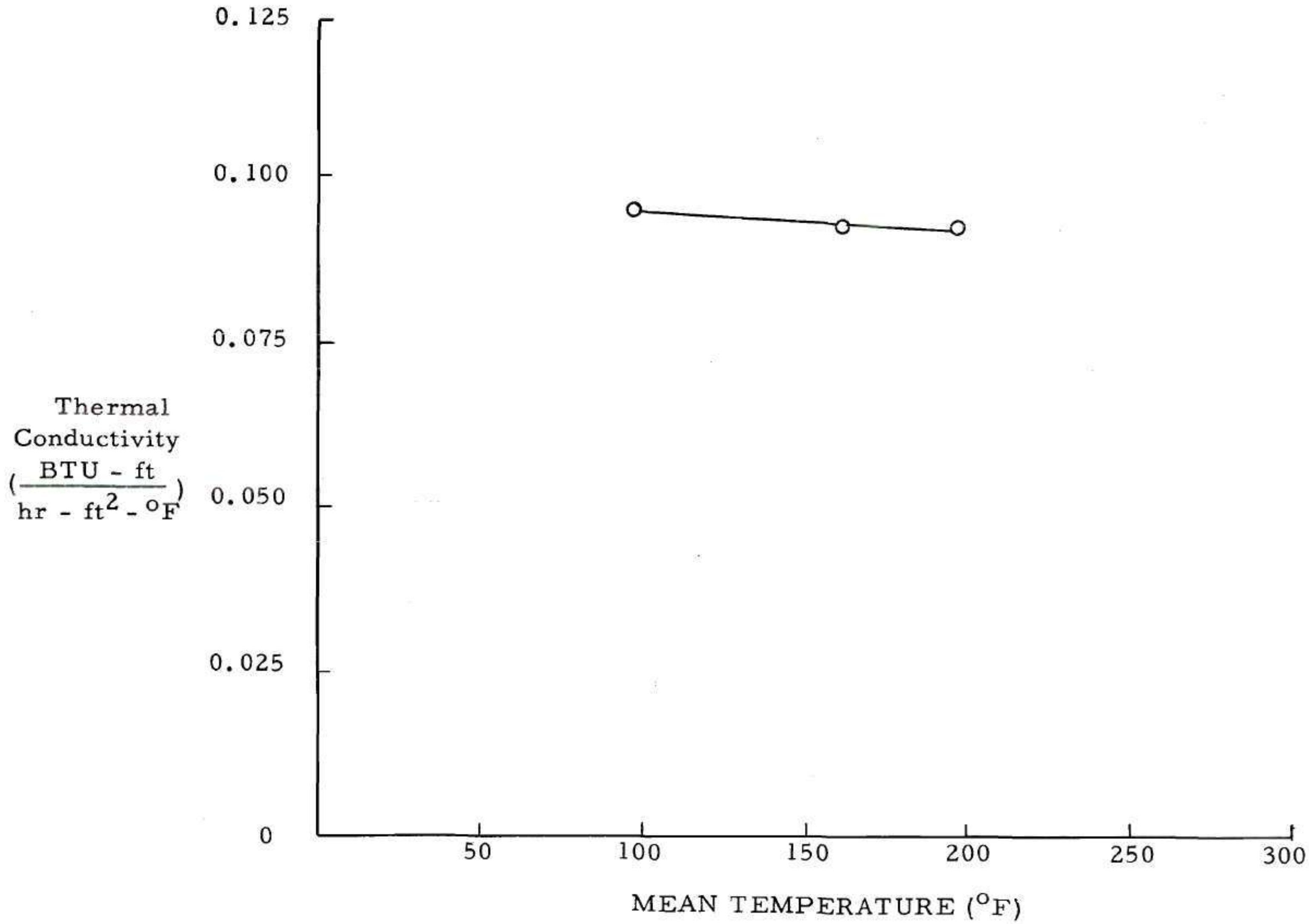

Figure 7. Thermal Conductivity of a Mineral Oil Specimen.

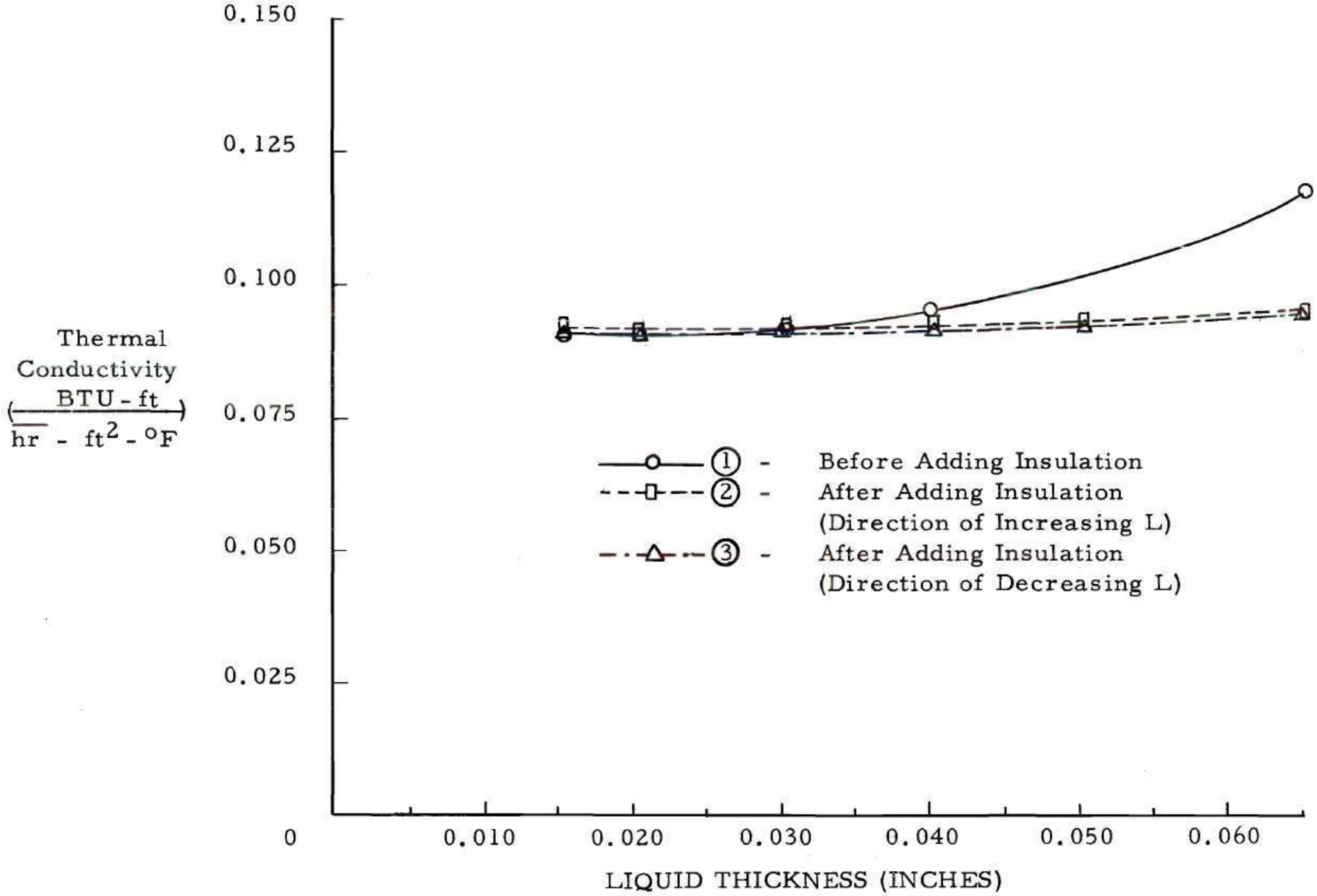

Figure 8. Comparison of the Thermal Conductivity of Mineral Oil Before and After Adding Insulation.

CHAPTER VI

CONCLUSIONS

The results of the performance tests indicate that good results can be obtained with the device at liquid thickness of 0.015 inches. The fact that negligible interaction occurred between thermocouples and between the guard and heater circuits is attributed to the method of insulating the resistance windings and the thermocouples. Operation of the equipment was found to be time consuming. Approximately 11 hours were necessary to obtain data for each point, as shown in the representative test of Run number 5, page 44. The data obtained, however, are considered to be within the prescribed limits of accuracy for liquid thickness up to 0.030 inches. At thicknesses of more than 0.030 inches, considerable increases in the measured values of thermal conductivity occurred.

In view of the results of the tests of the thermal conductivity vs. liquid thickness, and the investigation of Appendix D, it is probable that a part of the increase in K_m at the higher values of liquid thickness was caused by convective currents, originating in the retaining cup, which were manifest in the liquid gap at thicknesses in the range of 0.030 inches or greater. There is also evidence that the heat flux lines tended to bend away from a normal between the main heater plate and the cold plate in the vicinity of the ring guard insulator which caused an additional loss of heat in the test area. This loss was probably accentuated by increasing liquid thickness.

No definite conclusions were reached concerning this part of the experiment. The evidence presented was considered insufficient for proper analysis.

CHAPTER VII

RECOMMENDATIONS

The scope of the apparatus was limited by a maximum heater temperature of approximately 300°F, which was sufficient for the present experiment. The unit could probably perform satisfactorily at much higher temperatures if the Micarta insulator between the main heater and the upper guard heater were replaced with a material capable of withstanding higher temperatures.

As demonstrated in Appendix B, page 29 the accuracy of the device could be somewhat enhanced by the use of more accurate power measuring techniques. A satisfactory method would be to supply the main heater circuit with regulated direct current voltage and to place a calibrated shunt resistor in the circuit. A relatively inexpensive Leads and Northrup volt-box could be used to reduce the voltage drop across the shunt resistor and the main heater windings by a known amount, making possible the use of a sensitive and highly accurate galvanometer in voltage and current measurements.

In view of the discussion of Appendix D, page 36 it is recommended that a permanent insulator be installed around the liquid retaining cup, and that additional experiments be conducted at liquid thicknesses of 0.030 inches and greater.

APPENDIX A

SAMPLE CALCULATION

The equation used to calculate the thermal conductivity is

$$K_m = C \frac{E_h IL}{A(\Delta T)_3}$$

The following sample calculation is given for point Number 1 of Run number 5, page

$$(\Delta T)_3 = T_h - T_c = 107.5^{o}F - 71.5^{o}F = 36^{o}F$$

$$E_h = 4.3 \text{ volts}$$

$$I = \frac{E_r}{3} = \frac{7.23}{3} = 2.41 \text{ amperes}$$

$$L = 0.015 \text{ inches} = .00125 \text{ feet}$$

$$A = \frac{\pi}{4} D^2 = \frac{3.14}{4} \left(\frac{1.055}{12}\right)^2 = .01309 \text{ feet}^2$$

$$T_m = 89.5^{o}F$$

$$K_m = 3.413 \frac{(4.3)(2.41)(.00125)}{(.01309)(36)}$$

$$= .094 \text{ BTU per ft}^2 \text{ per hr per } {}^{o}F \text{ per ft}$$

APPENDIX B

ESTIMATED MAXIMUM ERROR

The equation for calculating thermal conductivity is

$$K_m = C \frac{EIL}{A(\Delta T)_3}$$

If the power terms, E and I, are considered as one variable, the total derivative of this equation is

$$dK_m = \frac{\partial K_m}{\partial (EI)} \; d(EI) \; \frac{\partial K_m}{\partial L} \; dL \; \frac{\partial K_m}{\partial A} \; dA \; \frac{\partial K_m}{\partial (\Delta T)} \; d(\Delta T)$$

$$dK_m = \frac{CL}{A\Delta T} \; d(EI) + \frac{C(EI)}{A\Delta T} \; dL + (-1) \; \frac{C(EI)L}{A^2 \Delta T} \; dA + (-1) \frac{C(EI)L}{A(\Delta T)^2} d(\Delta T)$$

Substituting into this expression and factoring yields:

$$dK_m = K_m \left[\frac{d(EI)}{(EI)} + \frac{dL}{L} - \frac{dA}{A} - \frac{d(\Delta T)}{T} \right]$$

$$\frac{dK_m}{K_m} = \frac{d(EI)}{(EI)} + \frac{dL}{L} - \frac{dA}{A} - \frac{d(\Delta T)}{T}$$

The error computed in this manner neglects possible heat losses or gains through the ring guard and upper guard. These errors may be accounted for as follows:

$$(EI)_1 = 2\pi K_1 C T_1 \quad \frac{(\Delta T)_1}{\ln r_1 - \ln r_2}$$

$$(EI)_2 = C \frac{K_2}{T_2} A_2 (\Delta T)_2$$

where the subscripts 1 and 2 refer to the ring guard and upper guard respectively, and the quantities to the right of the equality sign are as below:

K_1	=	Thermal conductivity of Teflon insulator = 0.2
T_1	=	Thickness of Teflon insulator = 0.37 inches
$(\Delta T)_1$	=	Temperature difference between ring guard and main heater = 0.25°F
r_1	=	Inside radius of Teflon insulator = 1.5 inches
r_2	=	Outside radius of Teflon insulator = 1.75 inches
K_2	=	Thermal conductivity of Micarta insulator between main heater and upper guard heater = 0.2
T_2	=	Thickness of Micarta insulator = 0.25 inches
A_2	=	Area of Micarta insulator
$(\Delta T)_2$	=	Temperature difference between main heater and upper guard heater = 0.25°F
C	=	Dimensional coefficient = 3.413 BTU per watts

Substitution of these values into the equations yields:

$$(EI)_1 = 2(3.14)(0.2)(3.413)(0.37) \frac{0.25}{\ln 1.5 - \ln 1.75} = .0009 \text{ watts}$$

$$(EI)_2 = 3.413 \left(\frac{0.2}{.0208}\right)(.049)(0.25) = 0.0108 \text{ watt}$$

The values assigned above are considered to be constants. Therefore the terms $(EI)_1$ and $(EI)_2$ are constants.

The total error now becomes

$$\frac{dK_m}{K_m} = \frac{d(EI)}{(EI)} + \frac{dL}{L} - \frac{dA}{A} - \frac{d(\Delta T)_3}{(\Delta T)_3} \pm \frac{(EI)_1}{(EI)} \pm \frac{(EI)_2}{(EI)}$$

The error thus computed is a maximum when all the terms to the right add and when the power and temperature terms in the demoninator are a minimum. Thus, the estimated maximum error can be expressed as

$$\text{maximum error} = \left|\frac{d(EI)}{(EI)}\right| + \left|\frac{dL}{L}\right| + \left|\frac{dA}{A}\right| + \left|\frac{d(\Delta T)_3}{(\Delta T)_3}\right| + \left|\frac{(EI)_1}{(EI)}\right| + \left|\frac{(EI)_2}{(EI)}\right|$$

The minimum power level used in any test was 2.4 watts (see point 6, Run Number 5, page 44. The minimum temperature drop, $(\Delta T)_3$, was 4°F (see point 1, Run Number 2, page 42. Considering the term dA to be negligible, and substituting the above values of (EI) and $(\Delta T)_3$ into the expression for estimated maximum error yields:

$$\text{estimated maximum error} = \pm\left(\frac{.05}{2.42} + \frac{.00025}{.015} + \frac{.04}{4} + \frac{.0009}{2.42} + \frac{.0108}{2.42}\right) \times 100\%$$

$$\text{estimated maximum error} = \pm\ 4.88\%$$

The terms d(EI) and $d(\Delta T)$ are functions of instrument accuracy and human error. Because these errors depreciate as the denominator terms increase, the estimated maximum error decreases considerably at higher power levels and temperature drops.

APPENDIX C

ANALYSIS OF THE EFFECT OF DIFFERENTIAL EXPANSION IN COOLING ELEMENT ON LIQUID GAP

The initial run with the device was a test of thermal conductivity vs. mean temperature of a mineral oil specimen 0.015 inch thick. Data as recorded during this test appear as Curve 1 in Figure 9, page 35. As shown in the figure, the slope of the resulting curve was positive, which was contradictory to all published data on petroleum fractions.

Analysis of the design of the device indicated the possibility of differential expansion or contraction between the heater support mechanism and the cold plate in the event of temperature changes in the cold plate, causing a corresponding decrease or increase in the liquid gap. The aluminum piston, support rods and cold plate have an expansion coefficient of approximately 17.5×10^{-6} in per in per $^{\circ}F$, while the steel of the adjustment micrometer has a coefficient of approximately 7.34×10^{-6} in per in per $^{\circ}F$. Preliminary tests showed that with a liquid thickness of 0.015 inch, a temperature increase of approximately $100^{\circ}F$ occurred in the cooling element. Using the expansion coefficients indicated above, and the geometry of the cooling element and adjustment mechanism, the actual per cent of error introduced into the gap thickness by differential expansion was computed.

An average value for the slope of the thermal conductivity vs. mean liquid temperature of petroleum fractions was taken from the work of J. F. D. Smith (9). Suitable corrections for the curve slope and liquid thickness errors were made in the calculation of the thermal conductivity values shown in Curve 1. The result is represented by Curve 2 of Figure 9, and indicated the need for constant cooling element temperature. Cooling coils were added and later results indicated that the error was considerably reduced.

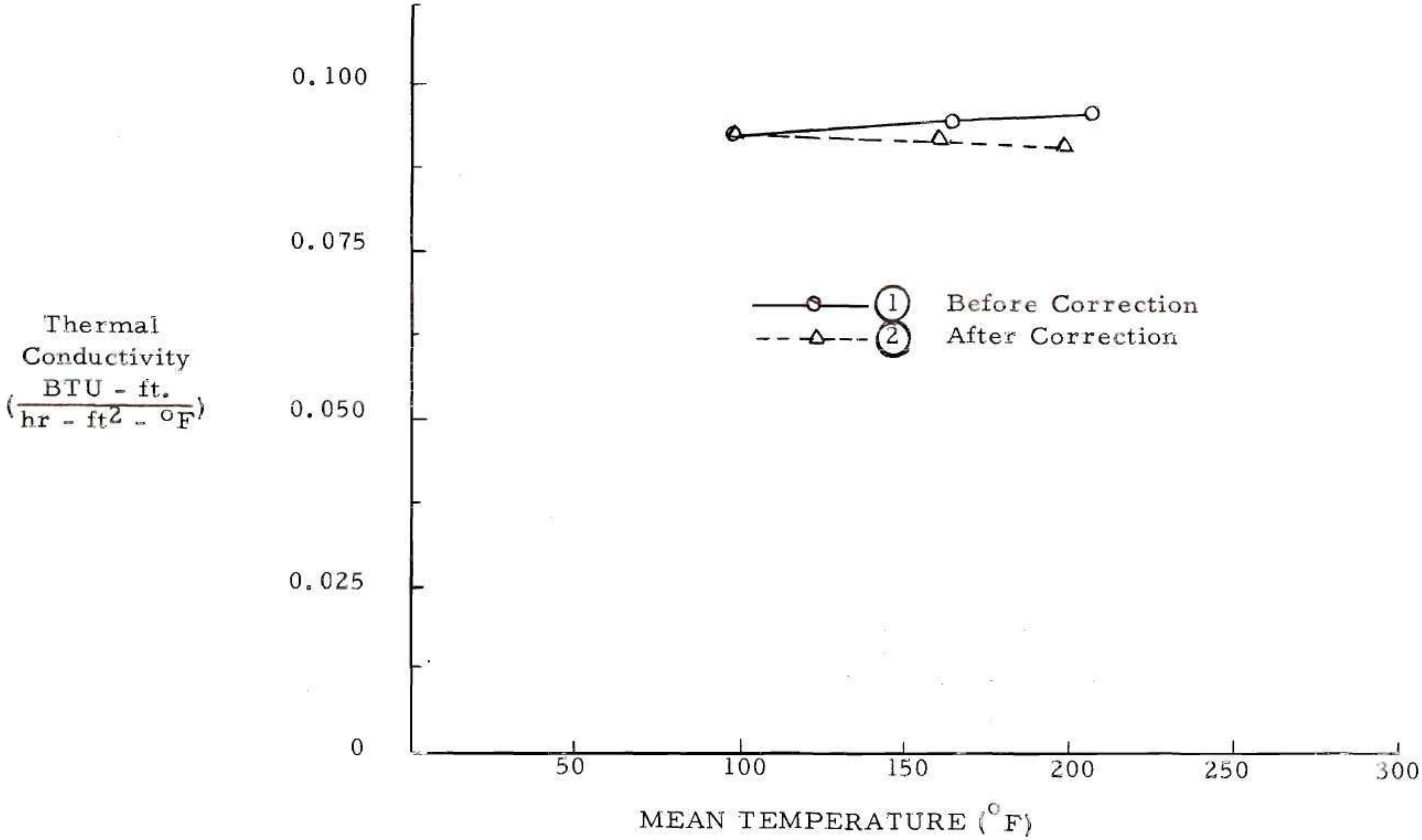

Figure 9. Comparison of the Measured Values of the Thermal Conductivity of Mineral Oil Before and After Correction For Error Caused by Differential Expansion.

APPENDIX D

INVESTIGATION OF POSSIBLE CAUSES OF THE INCREASED VALUE OF THE THERMAL CONDUCTIVITY OF MINERAL OIL AT FLUID THICKNESSES GREATER THAN 0.030 INCHES

Because of the small temperature differences maintained between the guard and heater circuits, it was valid to assume that the only appreciable avenue of heat loss from the main heater was through the fluid itself. Neglecting the heat input of the ring guard heater, a calculation of the possible heat loss from the main heater radially through the liquid layer was made for each liquid thickness by means of the conductive heat transfer relation.

$$EI = \frac{2\ \ CKL(\Delta T)}{\ln r_1 - \ln r_2}$$

where ΔT was taken as the difference in the mean liquid temperature and room temperature, L as the fluid thickness, r_1 as the main heater radius, and r_2 as the radius to the outside of the ring guard. The above calculation showed that a maximum heat loss of 0.7 per cent occurred at a liquid thickness of 0.065 inches, while the test data indicates that a maximum increase in K_m of 23.3 per cent occurred at the same point. It is readily apparent that radial heat loss could account for only a small part of the increase in K_m.

Another possibility of effective heat loss existed because of the Teflon insulator surrounding the ring guard. In examining this possibility, the following relatively valid assumptions were made.

1. The temperature of the isothermal plate and the main heater plate were equal and constant.

2. The temperature of the cold plate surface was constant.

3. The ring guard formed an effective adiabatic wall around the Teflon insulator.

Because of the difference in the values of K for copper and Teflon (approximately 220 and 0.2 respectively), the temperature of the liquid contact surface of the copper and Teflon are not the same. If the Teflon surface is at a lower temperature than that of the copper, the lines of heat flux bend in the direction of the lower temperature, causing an effective increase in the test area. Although the ring guard heat input substantially decreased the bending effect, it is probable that some heat loss occurred in this manner, especially at large values of liquid thickness. Figure 10, page 39 demonstrates the effect on the apparatus used in the present experiment of deviation of heat flux lines from a normal between the heater and cold plate surfaces.

The possibility of convective effect was also considered. Since theoretically, no convection should occur in fluids heated from above, the instrument was examined to determine if external convection forces could affect the liquid in the liquid gap. In the test apparatus, the liquid extended approximately 1/4 inch outside the heater and ring guard plate. The heat input to this liquid was from the ring guard in a radial direction. Since the fluid was cooled from both the top and bottom, a temperature gradient developed which peaked at some point within the liquid in the cup. The possibility therefore existed that convective motion occurred in this area, which could, at sufficient gap width, induce motion of the liquid within the gap. To minimize this possibility, additional insulation was added to the liquid retaining cup and Plexiglas cover.

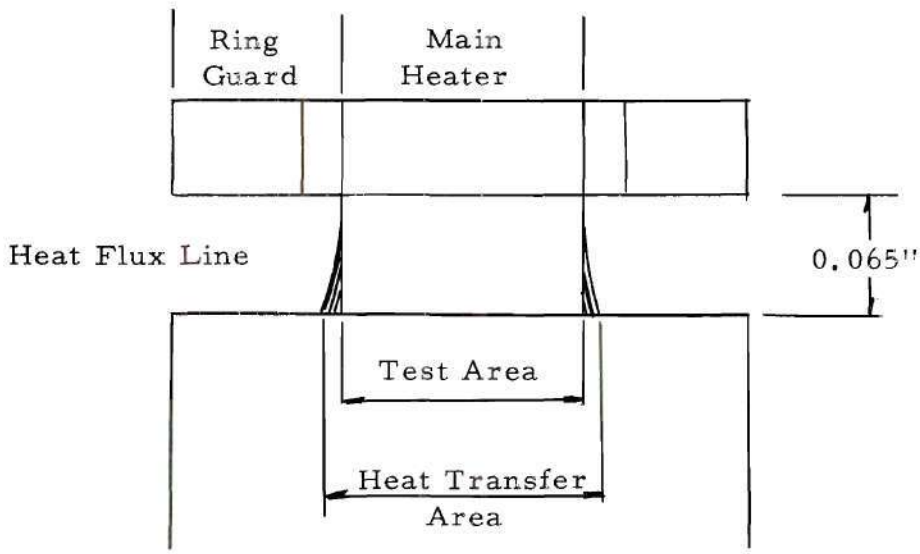

Figure 10. A Schematic Representation of the Effect of Deviation of Heat Flux Lines from a Normal Between the Heater Plate and the Cold Plate.

APPENDIX E

NOMENCLATURE

The following symbols were used in tabulating data:

$(\Delta T)_1$ = Temperature difference between ring and guard heater

$(\Delta T)_2$ = Temperature difference between upper guard and heater

$(\Delta T)_3$ = Temperature drop through test liquid

T_c = Temperature of cold plate

T_h = Temperature of main heater

T_m = Mean temperature of test liquid

E_h = Voltage drop across main heater windings

E_r = Voltage drop across calibrated resistor

I_r = Current in ring guard windings

I_{ug} = Current in upper guard windings

L = Test liquid thickness

K_m = Coefficient of thermal conductivity at the arithmetic mean temperature $\frac{T_c+T_h}{2}$

C = Dimensional coefficient

r_1 = Radius of main heater

r_2 = Radius to outside edge of ring guard

β = Cubic coefficient of thermal expansion

APPENDIX F

TABULATION OF EXPERIMENTAL DATA

Run Number 1. Thermal Conductivity of Mineral Oil vs. Mean Liquid Temperature Before Adding Cooling Coils.

Point Number	T_1	T_2	T_3	T_c		T_h	E_h	E_r	I_r	I_{ug}	L
	V	V	MV	MV	°F	MV	Volt	Volt	Amp	Amp	Inches
1	-5	2	0.297	1.187	85.7	1.484	2.15	3.38	1	0.3	0.015
2	-4	5	0.777	2.666	148.1	3.443	4.1	6.59	2.4	0.5	0.015
3	3	-5	1.329	3.536	183.0	4.865	5.34	8.6	3	1.0	0.015

Run Number 2. Thermal Conductivity of Water vs. Mean Liquid Temperature

Point Number	T_1 V	T_2 V	T_3 MV	T_c MV	T_c °F	T_h MV	E_h Volt	E_r Volt	I_r Amp	I_{ug} Amp	L Inches
1	3	-4	0.091	1.012	78	1.103	2.83	4.51	2.20	0.75	0.015
2	-4	-4	0.366	0.994	77.2	1.360	5.61	9.15	3.50	1.30	0.015
3	5	2	0.834	0.990	77	1.824	8.60	14.00	5.60	2.00	0.015
4	-2	4	1.011	2.226	130	3.237	9.50	15.40	6.75	2.75	0.015
5	4	5	2.003	2.154	127	4.157	13.30	21.65	8.20	3.2	0.015

Run Number 3. Thermal Conductivity of Castor Oil vs. Mean Liquid Temperature

Point Number	T_1 V	T_2 V	T_3 MV	T_c MV	T_c °F	T_e MV	E_h Volt	E_r Volt	L_r Amp	I_{ug} Amp	L Inches
1	-2	-5	0.115	1.200	86.2	1.315	1.75	2.84	1.2	0.5	0.015
2	4	1	1.013	1.240	88.0	2.253	5.12	8.20	4.0	1.2	0.015

Run Number 4. Thermal Conductivity of Mineral Oil vs. Mean Liquid Temperature, Including Representative Time Elements

Time	Point Number	T_1	T_2	T_3	T_c		T_h	E_h	E_r	I_r	I_{ug}	L
		V	V		MV	°F	MV	Volt	Volt	Amp	Amp	Inches
8:00				0.423	0.499	55.0	0.922	4.3	7.23	2.00	0.25	0.015
9:00		240	168	0.755	0.698	64.0	1.453	4.3	7.23	2.00	0.25	0.015
10:00		204	142	0.841	0.722	65.1	1.563	4.3	7.23	2.00	0.25	0.015
11:00		152	106	0.930	0.747	66.2	1.617	4.3	7.23	2.00	0.25	0.015
12:00		126	92	0.986	0.816	69.3	1.682	4.3	7.23	2.00	0.25	0.015
							Readjust L to account for change in TC					
1:00		124	88	0.868	0.789	68.1	1.657	4.3	7.23	2.25	0.35	0.015
2:00		46	38	0.813	0.835	70.6	1.678	4.3	7.23	2.60	0.55	0.015
3:00		-32	-20	0.830	0.883	72.2	1.713	4.3	7.23	2.50	0.5	0.015
4:00		-12	120	0.827	0.877	72.0	1.704	4.3	7.23	2.50	0.52	0.015
5:00		-6	5	0.829	0.870	71.7	1.699	4.3	7.23	2.50	0.52	0.015
6:00		-5	4	0.829	0.868	71.6	1.697	4.3	7.23	2.50	0.52	0.015
7:00	1	-5	3	0.827	0.865	71.5	1.692	4.3	7.23	2.50	0.52	0.015
							Total elapsed time 11 hours					
	2	-2	4	4.182	.967	76.0	5.149	9.3	15.4	3.25	1.25	0.015
	3	0	-5	5.698	1.040	79.2	6.738	10.8	17.6	5.20	1.80	0.015

Run Number 5. Thermal Conductivity of Mineral Oil vs. Liquid Thickness Before Adding Insulation to Liquid Cup and Plexiglas Cover

Point Number	T_1	T_2	T_3	T_c		T_h	E_h	E_r	I_r	I_{ug}	L
	V	V	MV	MV	°F	MV	Volt	Volt	Amp	Amp	Inches
1	-2	5	1.684	0.832	70.0	2.516	3.55	5.65	2.30	1.35	0.015
2	3	0	1.608	0.854	71.0	2.462	2.96	4.76	2.00	1.10	0.020
3	4	-2	1.576	0.890	72.6	2.476	2.45	3.84	1.80	1.00	0.030
4	-4	-4	1.527	0.865	71.5	2.392	2.10	3.40	1.65	0.70	0.040
5	3	2	1.493	0.899	73.0	2.393	1.95	3.05	1.50	0.50	0.050
6	-2	-3	1.308	0.990	77.0	2.298	1.67	2.67	1.25	0.35	0.065

Run Number 6. Thermal Conductivity of Mineral Oil vs. Liquid Thickness After Adding Insulation to Liquid Cup and Plexiglas Cover (Data Taken in Direction of Increasing Liquid Thickness)

Point Number	T_1 V	T_2 V	T_3 MV	T_c MV	T_c ^oF	T_h MV	E_h Volt	E_r Volt	I_r Amp	I_{ug} Amp	L Inches
1	1	-5	1.673	0.877	72.0	2.550	3.55	3.65	1.75	1.30	0.015
2	5	3	1.558	0.865	71.5	2.453	2.96	4.76	1.50	1.00	0.020
3	-5	2	1.568	0.899	73.0	2.467	2.45	3.84	1.10	0.95	0.030
4	4	-5	1.527	0.886	72.4	2.413	2.10	3.40	0.80	0.70	0.040
5	-3	5	1.601	0.890	72.6	2.491	1.95	3.05	0.65	0.60	0.050
6	-2	1	1.531	0.863	71.4	2.394	1.67	2.67	0.60	0.25	0.065

Run Number 7: Thermal Conductivity of Mineral Oil vs. Liquid Thickness After Adding Insulation to Liquid Cup and Plexiglas Cover (Data Taken in Direction of Decreasing Liquid Thickness)

Point Number	T_1	T_2	T_3	T_c		T_h	E_h	E_r	I_r	I_{ug}	L
	V	V	MV	MV	°F	MV	Volt	Volt	Amp	Amp	Inches
1	4	-5	1.666	0.902	73.2	2.568	3.55	5.65	1.80	1.25	0.015
2	2	-4	1.581	0.922	74.0	2.503	2.96	4.76	1.45	1.05	0.020
3	-4	2	1.557	0.886	72.4	2.443	2.45	3.84	1.00	0.90	0.030
4	-5	-3	1.552	0.894	72.8	2.446	2.10	3.40	0.80	0.75	0.040
5	5	5	1.578	0.868	71.6	2.446	1.95	3.05	0.65	0.65	0.050
6	1	-4	1.775	0.863	71.4	2.638	1.67	2.67	0.60	0.25	0.065

BIBLIOGRAPHY

1. Sakiadis, Byron C. and Jesse Coates, Studies of Thermal Conductivity of Liquids. Part II, Engineering Experiment Station Bulletin No. 35, Louisiana State University, Baton Rouge, Louisiana, 1953.

2. Ibid., p. 41.

3. Ibid., p. 10.

4. Ibid., p. 2.

5. Ibid., p. 41.

6. Smith, J. F. D., "The Thermal Conductivity of Liquids," Transactions, American Society of Mechanical Engineers, Vol. 58, (1936), p. 725.

7. Sakiadis and Coates, op. cit., p. 13.

8. Bates, Oscar K., "Thermal Conductivity of Liquids," Industrial and Engineering Chemistry, Vol. 28, (1936), p. 494.

9. Smith, J. F. D., "The Thermal Conductivity of Liquids," Industrial and Engineering Chemistry, Vol. 22, (1930), p. 1246.

10. Kaye, G. W. and W. F. Higgins, "The Thermal Conductivity of Certain Liquids," Proceedings of the Royal Society of London, Series A, Vol. 117, (1928), pps. 459-461.

11. Jakob, Max, Heat Transfer, Vol. 1, New York: John Wiley and Sons, Inc., 1953, pps. 161-163.

12. Maxwell, J. B., Data Book on Hydrocarbons. New York: D. Van Nostrand Company, Inc., 1950, p. 214.

13. Eckert, E. R. G., Introduction to the Transfer of Heat and Mass. New York: McGraw-Hill Book Company, Inc., 1950, p. 271.

14. Kaye and Higgins, op. cit., p. 468.